Call- and Put-Options

As you possibly have learned, the holder of a forward contract is obliged to trade at maturity. Unless the position is closed before maturity the holder must take possession of the asset, regardless of whether the underlying asset has risen or fallen in price. Wouldn’t it be nice if we only had to take possession of the asset if it had risen?

To address this wish derivatives known as options are traded. The two most famous ones are call options and put options. In this article we discuss first call options, later put options.

**Definition (Call Option)**
A call option gives the investor the right (not the obligation!) to buy an underlying asset at an agreed upon price (the strike price) at a date in the future (the expiration date)

Before we discuss a Call Option in detail we give some Option Terminology:

- **Premium** – the amount paid to the seller at the time of agreement. For a Call Option this premium is called $\text{Call}(K,T)$, for a Put Option (which will be discussed further below) the premium is called $\text{Put}(K, T)$.
- **Strike price ($K$)** – the amount for which the underlying asset can be bought (Call Option) or sold (Put Option)
- **Exercise** – the act of paying the strike price to receive/sell the asset
- **Time to expiration ($T$)** – time units (a time unit has the length t) until expiration
- **European Option** – the holder can exercise this option only at expiry
- **American Option** – the holder can exercise this option at any time during the life of the option (The right to exercise at any time is clearly valuable. Therefore the value of an American option cannot be less than an equivalent European option)
- **Market Price, or Spot Price ($S$)** – the current price you have to pay in the market for the Option.

Now, if you hold a call option, and at expiration of the option the price of the underlying asset $S$ is below the Strike Price $K$, the option is clearly worthless for you. It makes no sense to buy the asset for the higher strike price $K$, if you can turn around and buy it for the lower market price $S$.

But if the underlying asset has a market price that is above the Strike Price, things are different. Here you gain by the difference of the market price and the Strike Price. Why? You can buy the underlying asset for the Strike Price. You don’t have to purchase the asset at the higher market price. So you can save the difference of the market price and strike price.

We can summarize, that the holder of a call option wants the underlying asset to rise as much as possible so that he can buy the asset for a relatively small amount, then sell it and make money.
Let us formalize things a little bit: By viewing our purchased call option through payoff- and profit diagrams we get a better understanding about their worth at expiration date.

While a payoff diagram simply graphs the cash value at any point in time during the lifetime of the option, a profit diagram shows us exactly what we have earned from the purchase of the option. We just shift the payoff graph (orange line) downwards by the accumulated premium (accumulated by the risk free rate) at time $T$ (the so called “future value of the premium of the call option at time $T$). By doing this we arrive at the profit graph (dark blue line)

Exhibit C.1: The payoff- and profit-diagram of a purchased call option

The payoff of a purchased call option is given by the expression $\max(S - K, 0)$ and by including the future value (FV) of the purchase price we arrive at this expression for the profit of a purchased call: $\max(S - K, 0) - FV[Call(K, T)]$

The call option we have discussed so far is a purchased call option. But we can sell a call option also. Then we speak of a written call option.

The payoff and the profit of a written call option are just the mirror images of the corresponding purchased option. So the payoff- and profit-graphs of a written call option looks like
Exhibit C.2: The payoff- and profit-diagram of a written call option

The payoff of a written call option is given by: \( -\max(S - K, 0) \) and the profit of a written call by: \( -\max(S - K, 0) + FV[\text{Call}(K, T)] \).

In the derivative market we have the following naming convention: The purchased call option is named long, the written call option is named short. **Generally we can say that we have a derivative long, when the profit rises when the spot price rises too. And accordingly, we speak of a short position in a derivative, when the profit rises while the spot price declines.**

Now let us look at a simple strategy involving a call option: Imagine that you hold an asset. If you now write a call option on that asset you are doing what is known as **covered call** writing. It is “covered” because you already own the asset. If the call option is exercised then you just hand over the asset. By writing this option you gain the premium. You will not lose from this position: the worst that can happen is that the asset price rises and you have missed out the profits you would otherwise have made. If, on the other hand, the asset price falls you have taken in the premium.

By analyzing this position you see that selling covered calls has the effect of “giving away the upside”. That is, the option seller cannot benefit from increases in the asset price beyond the strike.

Now let us compare the profit of a purchased call option with the profit from a long forward. The long forward (red line in Exhibit C.3) shows that you lose substantially when the Spot Price is low. How can we avoid this unfavorable situation? We can purchase a call option. The profit of the purchased call option rises when the Spot Price is above the strike price (at the right of the black line), like the long forward rises if the Spot Price is greater than the agreed upon Forward Price. But the profit of the call is a little bit lower.
Let us look at the left of the black line. Here we have the unfavorable situation that we lose by holding the forward if the Spot Price declines. Here helps a call option: The “loose-situation” is leveled, meaning: our “loose-situation” doesn’t get worse as the blue line to the left of the (black) strike price $K$ shows. But for avoiding losing more and more money we have to pay a price, and this price is the future value of the premium of our call option. Remember, our long forward has no upfront payment unlike the purchased call, therefore the difference between the red and blue lines.

![Graph of a long forward and purchased call option]

**Exhibit C.3:** Purchased Call Option is Insurance

That means that a purchased call option protects against a falling asset price. So a purchased call option can be seen as insurance (where the insurance premium is the premium of the purchased call accumulated to the expiration time $T$)

Now let us talk about Put Options.

**Definition (Put Option)**

A Put Option gives the holder the right (but not the obligation!) to sell an underlying asset at an agreed-upon-price (the strike price) at a date in the future (the expiration date $T$).
Exhibit C.4: The payoff- and profit-diagram of a Purchased Put Option

If you believe that the underlyer is going to fall, then you really should buy a Put Option b/c in this scenario you can sell the underlyer for more than it is worth.

The payoff function of a Purchased Put Option is $\max(K - S, 0)$, and the profit function is $\max(K - S, 0) - FV(\text{Put}(K, T))$.

Now let us turn to written put options.

Selling a put (or “writing a put”, what is the same) is a options strategy where an investor (the put seller) writes a put contract, and by selling this contract to the put buyer, the put seller has sold the right to sell the underlying asset at a specific price (the strike price). Thus, the put buyer now has the right to sell the asset to the put seller at the strike price $K$.

But because you (the put seller) will receive the premium in exchange for the commitment to buy the asset at the strike price, you have some chances to go home with a gain. But only when the put buyer doesn’t exercise the put option.

Here is the graphic:
Exhibit C.5: The payoff- and profit-diagram of a Written Put Option

In formulae we have for the payoff-function for a Written Put Option – \( \max(K - S, 0) \) and for the profit-function for a Written Put Option: \(-\max(K - S, 0) + FV(\text{Put}(K, T))\).

Be aware of the minus-sign: If for example the strike price is 40 and the spot price is 20, then the put seller has to buy the underlying asset for 40 (from the put-buyer) but can sell it at the market for only 20. So he loses 20. Therefore we have to apply the minus-sign.